Exercise

Information-Based Complexity

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Summer term 2015

Sheet 2

Return until 12.05.2015, 12:00, into the box of the work group (Building 48, 6. floor)

Exercise 1:

Show that for arbitrary subsets $A \subseteq l_\infty^n$ the following holds:

$$\mathrm{rad}(A) = \frac{1}{2}\mathrm{diam}(A).$$

Exercise 2:

Let G be the linear space of continuous functions f with $f(0) = 0$, i.e.

$$G = \{f \in C[0,1] : f(0) = 0\},$$

equipped with the norm $\|f\|_\infty = \max_{0\le t\le 1} |f(t)|$. For $\alpha \in [1,2]$ let A be defined by

$$A = \{f \in G : -1 \le f(t) \le \alpha - 1 \quad \forall t \in [0,1]\}.$$

a) Show that $\mathrm{diam}(A) = \alpha \cdot \mathrm{rad}(A)$ and $\mathrm{rad}(A) = 1$.

b) Let G be defined as above, the solution operator is for $F = A$ defined by $S(f) = f$ and $N(f) = (f(t_1), f(t_2), ..., f(t_n))$, where $t_i \in [0,1], i = 1, .., n, t_i \ne t_j$ $(i \ne j)$.

   Show that $d(N, y) = \alpha \cdot r(N, y)$ and $r(N, y) = 1$ for all $y \in N(F)$.

Exercise 3:

The problem of numerical differentiation is defined as follows:

Given are function values of f. Our aim is to approximate $f^{(k)}(t_0)$ in a given point t_0.

Formulate the problem of numerical differentiation as an IBC-problem, i.e., define F, G, S, and Λ in a suitable way. Is this a linear problem?