Exercise

Information-Based Complexity

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Summer term 2015

Sheet 3

Return until 19.05.2015, into the box of the work group (Building 48, 6. floor)

Exercise 1:

Let E be a normed space with norm $\|\cdot\|$. A set of subsets E_i of a normed space E is called a cover (Überdeckung) of $F \subseteq E$, if $F \subseteq \bigcup_i E_i$. We define the ϵ–*entropy* H_ϵ of F as the smallest natural number k, such that there exists a cover of $F \subseteq E$ with at most 2^k subsets E_i with radius $r(E_i) \le \epsilon, i = 1, .., 2^k$.

Derive the ϵ–entropy for $D = [0,1]^d$ in the l_∞^n–norm.

Show that H_ϵ is the smallest number n of questions of type "$x_0 \in T$?", we have to ask to approximate x_0 with error $e_n \le \epsilon$, where $T \subseteq E$.

Exercise 2:

Let

$$F = \{f \in C[0,1]; \quad \|f\|_C \le 1\}, \quad G = C[0,1], \quad S : F \to G, \quad Sf = f.$$

Let N be the non-adaptive standard information operator, i.e.,

$$Nf = (f(t_1), ..., f(t_n)).$$

Prove that $r(N) = \frac{1}{2}d(N) = 1$.

Exercise 3:

We are given the following set F of functions $f : [0,1] \to \mathbb{R}$:

$$F = \{f = \chi_{[0,a]} : a \in [0,1]\},$$

where

$$\chi_{[0,a]}(t) = \begin{cases} 1 & : \quad t \le a \\ 0 & : \quad t > a \end{cases}.$$

We consider the integration problem

$$Sf = If = \int_0^1 \chi_{[0,a]} dt$$

on F with information of type $L_{t_0} f = \chi_{[0,a]}(t_0)$.
What is the minimal number of adaptive/non-adaptive information calls being necessary to approximate If with error $e_n \leq \epsilon$.