Exercise

Information-Based Complexity

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Sheet 4

Return until 02.06.2015, into the box of the work group (Building 48, 6. floor)

Exercise 1:

Let G be the space of continuous function $f : [0,1] \to \mathbb{R}$. The set A is defined by

$$A = \left\{ f \in G : \forall x, y \in [0,1] \quad |f(x) - f(y)| \leq |x - y| \quad \text{and} \quad \mathrm{f}(0) = 0, \mathrm{f}(1) = \frac{1}{2} \right\}$$

Show that A has exactly one center if G is equipped with the norm $\|f\|_2 = \left(\int_0^1 |f(t)|^2 dt \right)^{\frac{1}{2}}$. Moreover, show that it has infinitely many centers if G is equipped with the supremum norm,

$$\|f\|_\infty = \sup_{0 \leq t \leq 1} |f(t)|$$

Exercise 2:

a) Prove Smolyak's theorem in the case of $r(N) = 0$ (see slide 61).

b) Prove Smolyak's theorem in the case of $G = l_\infty^n$.

Exercise 3:

Let $a \in [0, +\infty)$. Find a linear problem defined by S, and two different information operators N_1, N_2 such that for all $n \in \mathbb{N}$

- $e_n(S, N_1) = a$,
- $e_n(S, N_2) = 0$.

Here, $e_n(S, N_i)$ denotes the n-th minimal error $e_n(S)$ where N_i is the admitted information operator.