Exercise

Information-Based Complexity

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Sheet 5

Return until 09.06.2015, 12:00, into the box of the work group (Building 48, 6. floor)

Exercise 1:

Let $0 < \alpha < 1$ and let F be the set of α–Hölder–continuous functions on $[0,1]$, i.e.

$$F = \{f \quad : \quad \forall s,t \in [0,1] : |f(s) - f(t)| \leq |s-t|^{\alpha}\} \quad .$$

We consider the integration problem $Sf = If = \int_0^1 f(t)dt$ on F with information $L_{t_0} = \delta(t_0) = f(t_0)$ (standard information).

Prove that $e_n(S) \asymp n^{-\alpha}$.

Exercise 2:

Given is the set F of Lipschitz–continuous functions on $[0,1]$, i.e.

$$F = \{f \quad : \quad \forall s,t \in [0,1] : |f(s) - f(t)| \leq |s-t|\} \quad .$$

We again consider the integration problem $Sf = If = \int_0^1 f(t)dt$ on F with information $L_{t_0}f = \delta_{t_0}f = f(t_0)$.

a) Prove that $e_n(S) = \frac{1}{4n}$.

b) Construct a quadrature formula that reaches this optimal error.

Exercise 3:

Let

$$Qf = \sum_{j=1}^{p} w_j f(\tau_j)$$

be a general quadrature formula for functions $f \in C[0,1]$, such that

$$\exists c > 0 : \forall f \in C^r[0,1] : |If - Qf| \leq c \max_{t\in[0,1]} |f^{(r)}(t)| \quad .$$

Show that the corresponding composite quadrature formula Q_n is quasi-optimal on $F = B_{C^r[0,1]}$, i.e. the following holds for Q_n:

$$e(Q_n) := \sup_{f \in B_{C^r[0,1]}} |If - Q_n f| = O(n^{-r}).$$