Exercise

Information-Based Complexity

Prof. Dr. S. Heinrich

Summer term 2015

Sheet 7

Return until 23.06.2015, into the box of the work group (Building 48, 6. floor)

Exercise 1:
Let $F_1 = G = C([0,1])$. We consider the approximation problem with the solution operator $Sf = f$ on $F = B_{C([0,1])}$ using

a) standard information;

b) linear information.

Show that in both cases $e_n(S) = 1$.

Exercise 2:
Let E, F, H be normed spaces and $m, n \in \mathbb{N}$. Show that the following properties for the Gelfand numbers hold:

a) $\forall S, T \in L(E, F) : c_{m+n}(S + T) \le c_m(S) + c_n(T)$

b) $\forall S \in L(E, F), \forall T \in L(H, E) : c_{m+n}(ST) \le c_m(S) \cdot c_n(T)$.

Exercise 3:
Let $0 < \alpha \le 1$, $F = \{f \in C^\alpha([0,1]) : \forall s, t \in [0,1] : |f(s) - f(t)| \le |s - t|^\alpha\}$ and $G = C([0,1])$. We consider the approximation problem with solution operator $Sf = f$ on F using

a) standard information;

b) linear information.

Show that in both cases $e_n(S) \asymp n^{-\alpha}$.