The Course Information Based Complexity (TU Kaiserslautern)

Content in general:

Information-based complexity theory (IBC) aims at finding the minimally necessary effort which is required for the numerical solution of problems of analysis. On a theoretical basis lower bounds are established. Upper bounds are derived by either investigating the efficiency and optimality of known algorithms or by developing new algorithms which are optimal in this setting. In a number of situations this approach led to completely new types of algorithms. Within this framework we study basic numerical problems like integration, approximation, parametric integration, solution of integral and differential equations. We also analyze stochastic (or Monte Carlo) algorithms whose main feature is the use of randomness in the calculation process. Many highly complex applications, where traditional methods fail, can only be treated by stochastic techniques.

Contents of the Chapters:

Chapter 1) general theory of information based complexity, basic definitions, different types of information, adaptive/nonadaptive information, standard information and linear information, linear algorithms, radius of information

Chapter 2) deterministic complexity of numerical integration, upper bounds and lower bounds for the n-th minimal error, efficient/optimal algorithms

Chapter 3) deterministic complexity of approximation, upper bounds and lower bounds for the n-th minimal error in the case of standard information and in the case of linear information, Gelfand numbers

Chapter 4) Complexity of integration in the randomized setting, upper bounds and lower bounds for the n-th minimal error, classical Monte-Carlo methods and optimal Monte-Carlo methods using variance reduction

Chapter 5) complexity of integral equations, solvability and uniqueness, Nystrm method, two

grids method, upper bounds and lower bounds for the n-th minimal error

Chapter 6) complexity of differential equations, solvability and uniqueness, theorem of Picard-Lindelf, Gronwall‘s lemma, optimal algorithms, upper bounds and lower bounds for the n-th minimal error